CS106X Practice Midterm

Exam Facts:

When: Thursday, November 21st from 7:00 - 8:30 p.m.
Where: Hewlett 201

Coverage

The open-book, open-note, closed-computer exam covers everything up through trees (binary search trees, binomial heaps, tries, etc.), although I will not test anything on graphs and inheritance. The exam will emphasize material not tested on the first exam, so expect to be drilled on pointers, dynamically allocated memory, linked lists, hashing and hash tables, and trees. Understand going in that issues pertaining to **&** versus ***** versus **.** versus **->** are details that matter very, very much.

The practice exam presented here was given as a **three-hour final** several years ago, so it's **much longer** than anything you'll experience during your 90-minute exam. Nonetheless, each of these practice problems exercises something I feel is important, and you should expect to see problems like these on your midterm.

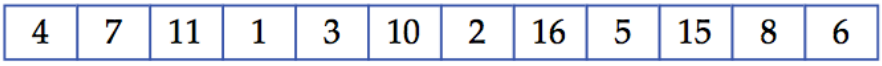
Note that your exam will include plenty of space to write your answers down, but I've excised all whitespace from this practice exam to save on paper. As with your first midterm, I'll include a summary of all the prototypes and interfaces I expect will be relevant.

1. Equivalence Classes	[10]	_____	_____
2. New Jersey Counties	[10]	_____	_____
3. Linked Lists and Sets	[10]	_____	_____
4. Tries and Removing Words	[8]	_____	_____
5. Patricia Tree Traversal	[7]	_____	_____
6. Short Answer	[5]	_____	_____
Total	**[50]**	_____	_____

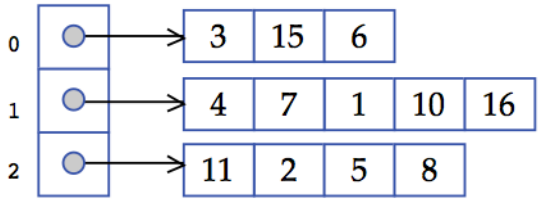
Problem 1: Equivalence Classes [10 points]

Two positive integers **m** and **n** are said to be modulo-**d** equivalent if and only if **m % d == n % d**—that is, both **m** and **n** produce the same remainder when divided by **d**. When two integers are modulo-**d** equivalent, we say they fall into the same *equivalence class*. Partitioning an array of positive integers into its modulo-**d** equivalence classes amounts to the creation of **d** new arrays and distributing the integers across them. Those integers modulo-**d** equivalent to **0** are placed in the 0th array, those modulo-**d** equivalent to **1** are placed in the next array, and so forth.

As an illustration, consider the following array of positive integers:

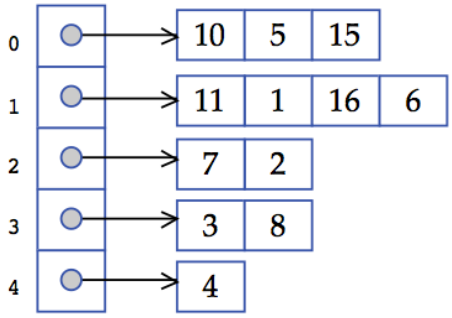


The three modulo-3 classes of this array could be represented by this array of three arrays:



Note the all integers of the original array are represented exactly once, and that the perfect multiples of 3 were placed in the 0th class, integers one more than a perfect multiple of 3 were placed in the next class, and those two more than a perfect multiple of three were placed in the final class. In general, an integer **n** is placed in the **k**th class whenever **n % 3** equals **k**.

The five modulo-5 classes of the original array would then be represented by this array of five arrays:



Again, the generalization here is that an integer **n** is placed in the **k**th array whenever **n % 5** equals **k**.

a. [4 points] First, implement the **computeClassSizes** function, which takes an array, the array's size, and an integer **d**, and computes all of the sizes of the modulo-**d** classes. The function should dynamically allocate space for an integer array of length **d**, and should then go on to write all of the class sizes to that array. The size of the modulo-0 class should be stored at index **0**, the size of the modulo-1 class should be stored at index **1**, and so forth. You may assume that all integer arguments are positive, so that no error checking is necessary. The base address of the dynamically allocated array should be placed at the address supplied through the last parameter. You should not use any objects or records at all, as the intent here to exercise your understanding of primitives and raw memory. Note that most of the points are dedicated to the memory directives and syntax—operator **new**, **&**, *****, **->**, etc.—that you've learned in CS106X.

```
static void computeClassSizes(const int array[], int size,
                              int d, int **classSizes) {
```

b. [6 points] Now leverage your **computeClassSizes** function to implement the **partitionIntoClasses** function, which distributes the integers of the specified array over the **d** classes. The array of classes is dynamically allocated, and each class is itself dynamically allocated to store the exact number of integers that fall into that class. Ultimately, two arrays needs to be returned, so the base address of each should be stored at the two locations specified via **classes** and **classSizes**. Again, the proper use of **&**, *****, **[]**, and so forth really matter here, so be very careful with them. Don't use any objects and records.

```
static void partitionIntoClasses(const int array[], int size, int d,
                                 int ***classes, int **classSizes) {
```

Problem 2: New Jersey Counties [10 points]

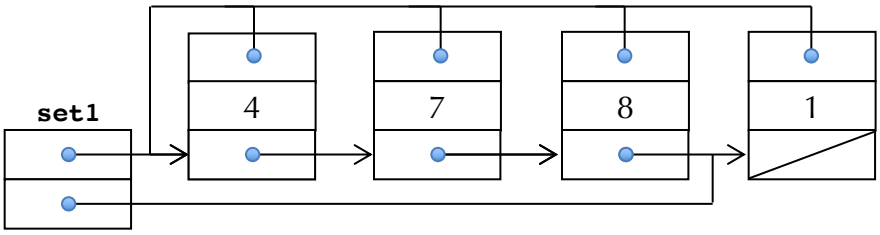
Analyze the following code snippet, starting with a call to **`somerset`**, and draw the state of memory at the point indicated—just before the **`bergen`** helper function returns. Be sure to differentiate between stack and heap memory, note values that have not been initialized, and identify if and where memory has been orphaned. Generate your final diagram on the next page, and feel free to tear this page out so you can easily refer to it.

```
struct county {
   int atlantic;
   county *burlington[4];
   county **hudson;
};

static void somerset() {
   county salem[3];
   salem[0].atlantic = 8;
   salem[1].burlington[0] = NULL;
   salem[1].burlington[1] = salem;
   salem[1].burlington[2] = &salem[1];
   salem[1].burlington[3] = salem[1].burlington[1];
   county **ocean = &(salem->burlington[1]);
   ocean = &ocean[1];
   salem[1].hudson = ocean;
   ocean[1] = new county;
   ocean[1][0] = salem[1];
   bergen(salem[1], ocean, ocean);
}

static void bergen(county& passaic, county **warren, county **& sussex) {
   warren = new county *;
   *warren = &passaic;
   passaic.hudson = warren;
   sussex = passaic.burlington;
   *&sussex = NULL;
                    ⇐ Draw the state of memory just before bergen returns.
}
```

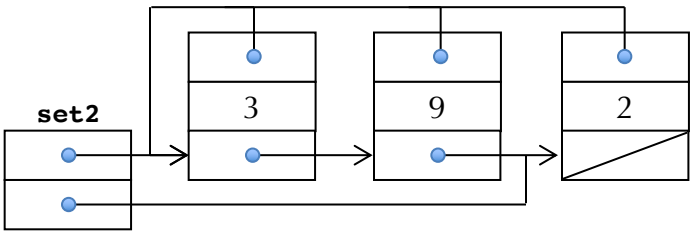
Problem 3: Linked Links and Disjoint Sets [10 points]

Assume an unsorted set of distinct integers is stored in a linked list, where each node in the list stores not only the address of its successor, but also the address of the list's front node. The set itself tracks the addresses of both the head and tail nodes of the list, much like the **`Queue`** template container does.

For example, the set {4, 7, 8, 1} would look like this:



and the set {3, 9, 2} might look like this:

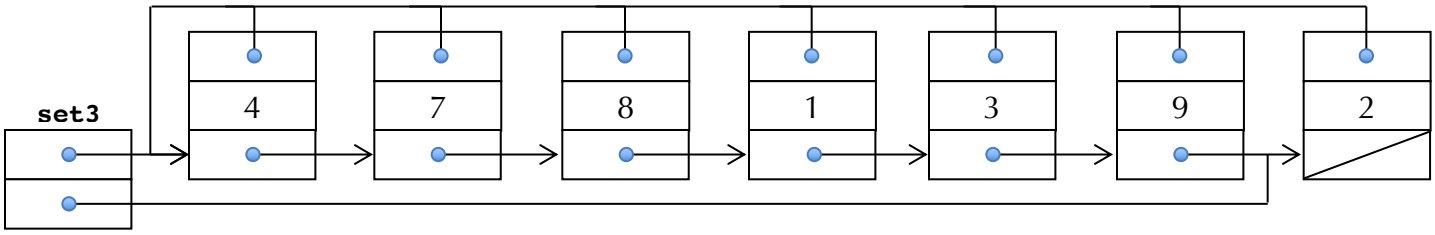


The data structures we'll use to support the above (we'll go with exposed **`struct`**s and not worry about encapsulation or object orientation) are as follows:

```
struct node {
   node *front;
   int value;
   node *next;
};
```

```
struct set {
   node *head;
   node *tail;
};
```

One nifty feature of our linked list approach is that set unioning is conceptually trivial—particularly if we assume that the two sets being unioned are nonempty and have no elements in common. If, for instance, **`set1`** and **`set2`** as drawn above were unioned into a third set, that third set would look like this:



In particular, the tail node of the first set is updated to point to the head node of the second, and all **`front`** fields in the second set's list are updated to point to the front node

of the first. No memory is allocated anywhere, as the nodes of the two original sets are donated to the third.

a. [5 points] Implement the **constructUnion** function, which accepts two sets by reference and returns their union (reusing existing nodes instead of allocating new ones). Assume the two incoming sets are nonempty and disjoint (e.g. no elements in common), and don't worry about cleaning up the two incoming sets in any way, as it's reasonable to assume that those two sets won't be used anymore, as their memory is being cannibalized and donated to a third one.

```
static set constructUnion(set& set1, set& set2) {
```

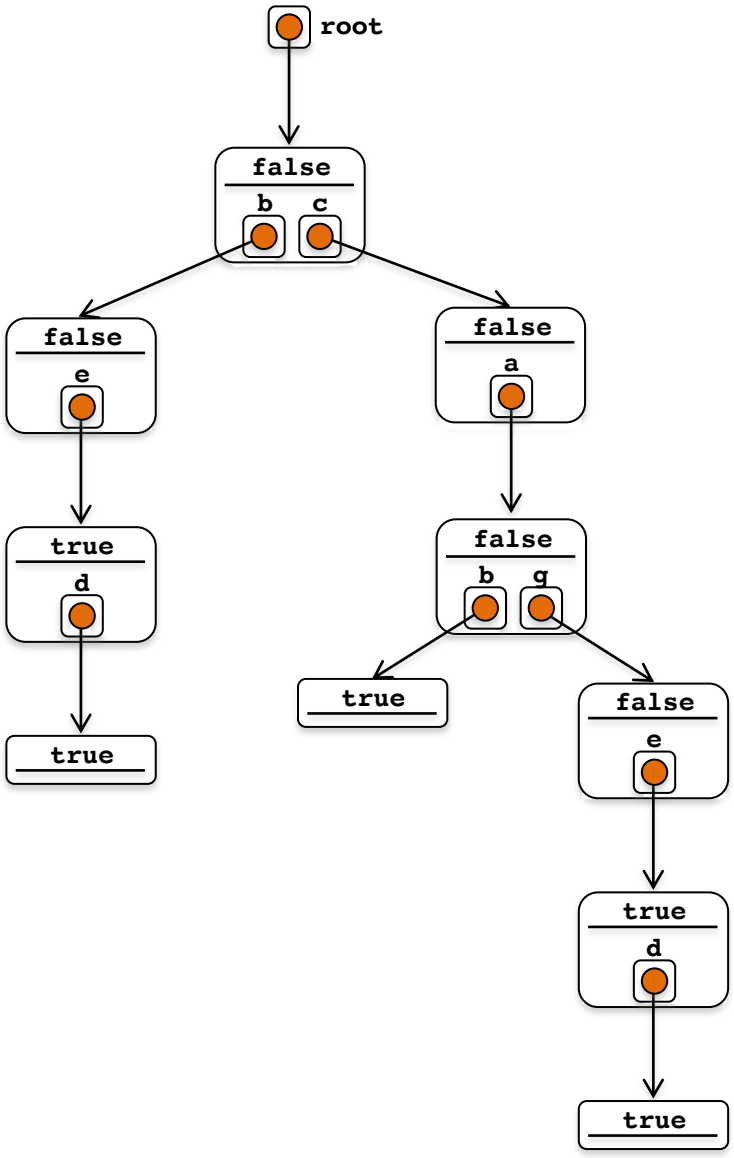
b. [5 points] Of course, one needs to be able to construct these sets in the first place. Implement the **buildSet** function, which accepts the provided **Vector<int>**—assumed to be nonempty and free of duplicates—and constructs and returns the equivalent set as described above. Recall that the linked list backing the set need not be sorted. You'll need to dynamically allocate nodes—one for each number in the referenced **Vector<int>**—and wire everything up as described above.

```
static set buildSet(Vector<int>& numbers) {
```

Problem 4: Tries and Removing Words [8 points]

Recall that the primary, object-oriented data structure we used to build a trie is defined as:

```
struct node {
   bool isWord;
   Map<char, node *> suffixes;
   node() { isWord = false; }
   ~node() { foreach (char ch: suffixes) delete suffixes[ch]; }
};
```



Implement the **`remove`** function, which removes the specified **`word`** from the trie, being careful to dispose of any nodes for prefixes that are no longer prefixes. To illustrate, let's inspect the sample trie I presented in lecture (presented on the right), which encodes five words: **`"be"`**, **`"bed"`**, **`"cab"`**, **`"cage"`**, and **`"caged"`**. If the word **`"be"`** is removed, then a single **`true`** if changed to **`false`** and that's that. No nodes are deleted, because all nodes relevant to **`"be"`** are needed to encode **`"bed"`**. If after removing **`"be"`** we remove **`"bed"`**, then we need to remove quite a few nodes, because **`"b"`**, **`"be"`**, and **`"bed"`** are no longer prefixes of anything. Additionally, the **`'b'`** entry in the root node's **`suffixes`** map would need to be removed as well.

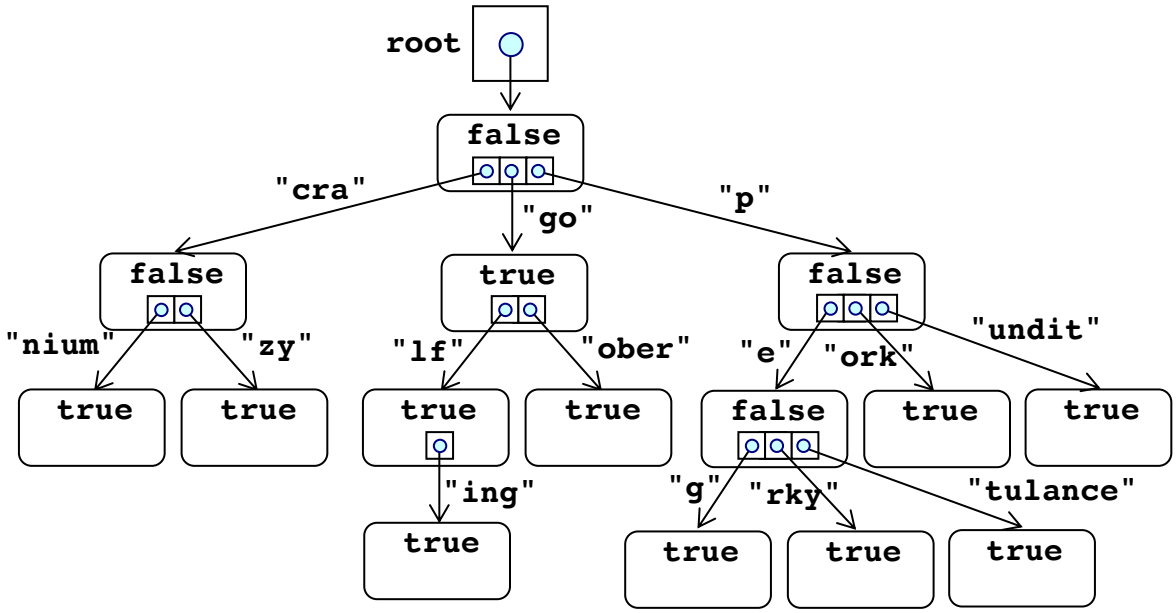
Special care must also be taken to see if the last remaining word is removed, in which case all remaining nodes would need to be killed off and the root would need to be set to **`NULL`**. Oh, and if the word isn't actually present, then you should just return without modifying the trie.

You may assume that the trie is always well formed in that there are no **`NULL`** pointer values in the **`Map`**s, and that all of the leaf nodes in the trie have their **`isWord`** fields set to **`true`**. Present your implementation of the **`remove`** function.

```
static void remove(node *& root, const string& word) {
```

Problem 5: Patricia Tree Traversal [7 points]

This problem leverages a data structure introduced in an earlier section handout, but the theory backing it is reproduced here in its entirety. Let's again consider the following illustration:



What's drawn above is an example of a **Patricia tree**—similar to a trie in that each node represents some prefix in a set of words. The child pointers, however, are more elaborate, in that they not only identify the sub-tree of interest, but they carry the substring of characters that should contribute to the running prefix along the way. Sibling pointers aren't allowed to carry substrings that have common prefixes, because the tree could be restructured so that the common prefix is merged into its own connection. By imposing that constraint, that means there's at most one path that needs to be explored when searching for any given word.

The children are lexicographically sorted, so that all strings can be easily reconstructed in alphabetical order. When a node contains a **`true`**, it means that the prefix it represents is also a word in the set of words being represented. [The root of the tree always represents the empty string.]

So, the tree above stores the following words:

cranium, *crazy*, *go*, *golf*, *golfing*, *goober*, *peg*, *perky*, *petulance*, *pork*, and *pundit*.

These two type definitions can be used to manage such a tree.

```
struct connection {
   string letters;
   struct node *subtree;          // will never be NULL
};

struct node {
   bool isWord;
   Vector<connection> children;  // empty if no children
};
```

Implement the **collectAllWords** function, which traverses the supplied tree as necessary and populates the referenced **Vector**—assumed to be empty starting out—so that by the end of execution it contains all of the tree's strings, sorted low to high. You should use recursion, and you'll want to implement this using a wrapper function.

```
static void collectAllWords(const node *root, Vector<string>& words) {
```

Problem 6: Short Answers [5 points]

a. [1 point] Assume some permutation of the numbers 1 through 3 is inserted into an initially empty binary search tree and assume that the binary tree doesn't self-balance. How many different permutations of the numbers 1 through 3 result in a perfectly balanced tree?

b. [1 point] What would the Huffman encoding tree for the string **`"bbbbaaacdcabbbb"`** look like? (Don't worry about the pseudo-EOF, and understand that there are many correct encoding trees, not just one.)

c. [1 point] Describe two key differences between a reference and a pointer (e.g. contrast **`int&`** to **`int*`**).

d. [1 point] Explain why the **`merge`** operation for binomial-heap-backed priority queues is so much faster than the **`merge`** operations for the unsorted-vector-, the sorted-doubly-linked-list-, and the binary-heap-backed priority queues.

e. [1 point] What was your favorite assignment? What did you like about it?